

UNITARITY IN HIGHER DIMENSIONS AND GAUGE UNIFICATION *

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Unitarity of the 4d standard model is ensured by the conventional Higgs mechanism with a fundamental spin-0 Higgs boson, responsible for gauge boson mass-generations. On the contrary, Kaluza-Klein (KK) compactification of extra spatial dimensions can geometrically realize the gauge boson mass generation without invoking a fundamental Higgs scalar. We reveal that massive gauge boson scattering in the compactified theories is unitary at low energies, and the unitarity violation is *delayed* to the intrinsic ultraviolet (UV) scale of the higher dimensional gauge theory. We demonstrate that this is a generic consequence of the “geometric Higgs mechanism” (GHM), manifested via Kaluza-Klein equivalence theorem (KK-ET). We further show that the presence of many gauge KK states below the UV cut-off scale imposes strong bounds on the highest KK level (N_{KK}). Applying these bounds to higher-dimensional SUSY GUTs implies that only a small number of KK states can be used to accelerate gauge coupling unification, and suggests that the GUT scale in the 5d minimal SUSY $SU(5)$ is above 10^{14} GeV.

1. The Puzzle: Unitarity in 4d versus Higher-d

Artists have explored extra dimensions since at least 1909¹, more than a decade before the first scientific 5d theory proposed by physicists Kaluza-Klein (KK)² who attempted to unify electromagnetism with Einstein gravity. When gauge bosons propagate in higher dimensional space, the compactification results in KK towers of *massive* vector bosons in 4d, with their masses characterized by the (inverse) size R of the extra dimensions. This

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provides a “geometric” realization of gauge boson mass generation, contrary to the conventional Higgs mechanism which invokes a fundamental spin-0 physical Higgs boson³.

A Higgsless Yang-Mills gauge theory can have a gauge boson mass term put in by hand, while perfectly respecting the gauge symmetry in the non-linear realization. The real problem is *Unitarity Violation*, i.e., the scattering of the longitudinal components of the massive gauge bosons has bad high energy behavior because the longitudinal polarization vector grows with energy, $\epsilon_L^\mu(k) = \frac{k^\mu}{m_v} + O\left(\frac{m_v}{E}\right)$, where m_v is the gauge boson mass. In consequence, a Higgsless standard model (SM) can only be an *effective theory* valid up to energies no higher than the unitarity violation scale,⁴ $E < \Lambda_U = \sqrt{8\pi} v \simeq 1.2 \text{ TeV}$. As we know, in the 4d SM it is the inclusion of a *physical* Higgs boson that cures the bad high energy behavior.

Now it is natural to ask: *Why would we expect the higher dimensional gauge theory to be unitary?* Consider first a non-compactified massless Yang-Mills theory in $D = 4 + \delta$ dimensions, which is nonrenormalizable because the gauge coupling has mass-dimension $\dim(\hat{g}) = -\delta/2$. Although the $2 \rightarrow 2$ scattering amplitude of D -dimensional *massless* gauge bosons behaves as constant of $O(\hat{g}^2)$, we note^{5,7} that the s -partial wave grows with energy $\sqrt{s} = E$ due to the D -dimensional phase space and is given by, in the $SU(k)$ gauge-singlet channel,^{5,7}

$$\hat{a}_{00} = \frac{E^\delta}{2(16\pi)^{1+\frac{\delta}{2}} \Gamma(1+\frac{\delta}{2})} \int_0^\pi d\theta (\sin \theta)^{1+\delta} \hat{T}_0 = \frac{2k\sqrt{\pi} \left[4(2+\delta) - \frac{\delta^3}{1+\delta}\right] (\hat{g} E^{\frac{\delta}{2}})}{(16\pi)^{1+\delta/2} \delta(2+\delta) \Gamma(\frac{1+\delta}{2})}. \quad (1)$$

Consequently, we find^{5,7} that the unitarity is violated at the intrinsic ultraviolet (UV) scale of $O(\hat{g}^{-2/\delta})$,

$$E < \Lambda_U = \left[\frac{(16\pi)^{1+\frac{\delta}{2}} \delta(2+\delta) \Gamma(\frac{1+\delta}{2})}{2k\sqrt{\pi} \left|4(2+\delta) - \frac{\delta^3}{1+\delta}\right|} \frac{1}{\hat{g}^2} \right]^{1/\delta}. \quad (2)$$

A realistic higher-dimensional theory will be compactified. *Why would a compactified gauge theory be unitary?* Note that the compactified gauge theory in 4d will contain towers of spin-1 *massive KK gauge bosons*, whose masses are generated by the “geometry” rather than by a fundamental Higgs boson. As explained earlier, the $W_L W_L$ scattering in the 4d SM is *non-unitary* without the Higgs contribution! So, the puzzle is: *why would such a compactified KK gauge theory be unitary at all?*

2. Geometric Higgs Mechanism & Unitarity of 5d Yang-Mills

We start by considering a generic 5d Yang-Mills theory of $SU(k)$, compactified on the orbifold $\mathbb{S}^1/\mathbb{Z}_2$. This corresponds to imposing the Neumann and Dirichlet boundary conditions (BCs) on a line segment $[0, L]$ ($L \equiv \pi R$), for the 5d gauge fields $(\hat{A}_\mu^a, \hat{A}_5^a)$ respectively,

$$\partial_5 \hat{A}_\mu^a \Big|_{x^5=0,L} = 0, \quad \hat{A}_5^a \Big|_{x^5=0,L} = 0. \quad (3)$$

Eq. (3) preserves the gauge symmetry $SU(k)$ in 4d, but we can construct other consistent BCs which reduces the rank of the gauge group, and in the simplest case we fully break $SU(k)$ by the following BCs,^{9,10,11}

$$\partial_5 \hat{A}_\mu^a \Big|_{x^5=0} = 0, \quad \hat{A}_\mu^a \Big|_{x^5=L} = 0; \quad \hat{A}_5^a \Big|_{x^5=0} = 0, \quad \partial_5 \hat{A}_5^a \Big|_{x^5=L} = 0. \quad (4)$$

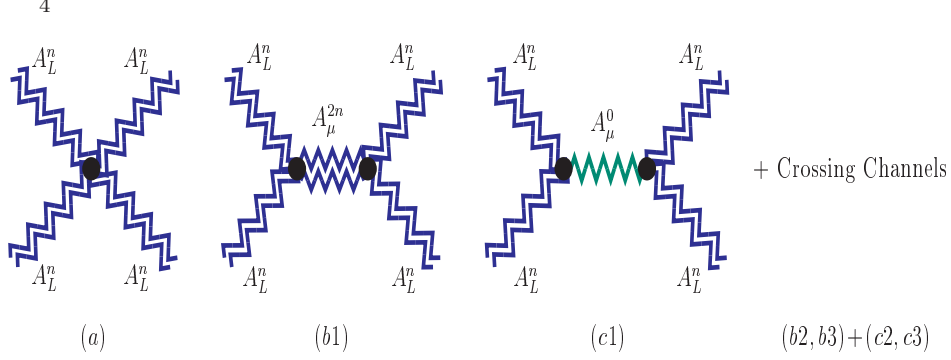
Thus we can derive Fourier expansions for $(\hat{A}_\mu^a, \hat{A}_5^a)$, and integrate out x^5 . Under (3), the resulting 4d KK Lagrangian contains the kinetic term,

$$\mathcal{L}_{\text{KE}} = -\frac{1}{4} \left[\left(\partial_{[\mu} A_{\nu]}^{a0} \right)^2 + \sum_{n=1}^{\infty} \left(\partial_{[\mu} A_{\nu]}^{an} \right)^2 \right] - \frac{1}{2} \sum_{n=1}^{\infty} [M_n A_\mu^{an} - \partial_\mu A_5^{an}]^2, \quad (5)$$

where $M_n = \frac{n}{R}$ is the mass of the KK state at level- n . It is important to note that (5) contains a mixing term $M_n A_\mu^{an} \partial^\mu A_5^{an}$, which enforces the dynamical conversion $A_5^{an} \longleftrightarrow A_L^{an}$ so that each KK state A_μ^{an} acquires a longitudinal component and becomes massive. Without invoking any extra *physical* Higgs boson, this is a *geometric* realization of gauge boson mass generation, which we call the “Geometric Higgs Mechanism” (GHM) and whose important consequences will be explored below (9). To eliminate the $A_\mu^{an} - A_5^{an}$ mixing in (5), we construct the general R_ξ gauge-fixing term,⁵

$$\mathcal{L}_{\text{GF}} = \sum_{n=0}^{\infty} -\frac{1}{2\xi} (F^{an})^2, \quad F^{an} = \partial^\mu A_\mu^{an} + \xi M_n A_5^{an}, \quad (6)$$

where ξ is the gauge-fixing parameter. The Faddeev-Popov ghost term \mathcal{L}_{FP} can be derived accordingly. Without physical Higgs boson one may naively expect, from the lesson of 4d Higgsless SM, that the scattering of longitudinal KK gauge bosons would violate unitarity at a scale $\Lambda_U \sim \frac{4\pi M_n}{g} = \frac{4n\pi}{gR} = \frac{4n\pi^{\frac{3}{2}}}{\hat{g}\sqrt{R}} = \frac{4n\pi^2 g}{\hat{g}^2}$, where $g = \hat{g}/\sqrt{\pi R}$. However, this cannot be true because by adjusting R to be arbitrarily large, the unitarity violation scale Λ_U would be arbitrarily below the intrinsic UV scale of $O(1/\hat{g}^2)$. A deeper reason is yet to be sought! We first compute the

Figure 1. Longitudinal scattering $A_L^{an} A_L^{an} \rightarrow A_L^{an} A_L^{an}$ in compactified KK theory.

longitudinal scattering $A_L^{an} A_L^{an} \rightarrow A_L^{an} A_L^{an}$ in Fig.1 and find the leading amplitude is only of $O(E^0)$,⁵

$$\mathcal{T}[A_L^{an} A_L^{bn} \rightarrow A_L^{cn} A_L^{dn}] = g^2 [C^{abe} C^{cde} T_1 + C^{ace} C^{dbe} T_2 + C^{ade} C^{bce} T_3],$$

$$T_1 = \frac{5}{2}c, \quad T_2 = -\frac{8c^2 - 5c + 9}{2(1-c)}, \quad T_3 = \frac{8c^2 + 5c + 9}{2(1+c)}, \quad (7)$$

where $c = \cos \theta$, and the small terms of $O(M_n^2/E^2)$ are ignored.

Diagram	T_1	T_2	T_3
(a)	$6c(\kappa^4 - \kappa^2)$	$\frac{3}{2}(3 - 2c - c^2)\kappa^4$ $-3(1-c)\kappa^2$	$-\frac{3}{2}(3 + 2c - c^2)\kappa^4$ $+3(1+c)\kappa^2$
(b1 \oplus 2 \oplus 3)	$-2c(\kappa^4 + \kappa^2)$	$-\frac{1}{2}(3 - 2c - c^2)\kappa^4$ $+3(1-c)\kappa^2$	$\frac{1}{2}(3 + 2c - c^2)\kappa^4$ $-3(1+c)\kappa^2$
(c1 \oplus 2 \oplus 3)	$-4c\kappa^4$	$(-3 + 2c + c^2)\kappa^4$ $-8c\kappa^2$	$(3 + 2c - c^2)\kappa^4$ $-8c\kappa^2$
Sum	$-8c\kappa^2$	$-8c\kappa^2$	$-8c\kappa^2$

In the above Table, we summarize the nontrivial $O(E^4)$ and $O(E^2)$ cancellations, with $\kappa \equiv E/(2M_n)$ and $E \equiv \sqrt{s}$, where the summed $O(E^2)$ terms are found to be exactly vanishing after using Jacobi identity $C^{abe}C^{cde} + C^{ace}C^{dbe} + C^{ade}C^{bce} = 0$. We also verified exact E -cancellations in all other channels^{5,7}. Hence, so long as the 4d gauge coupling is perturbative, the individual KK scattering channel is manifestly unitary!

The Geometric Higgs Mechanism (GHM) we observe essentially results from the 5d gauge symmetry and the proper compactification⁵. Based

on the 5d gauge symmetry (or the equivalent BRST invariance), we have extended the 4d derivation⁸ to deduce a 5d Slavnov-Taylor identity^{5,9},

$$\langle 0 | T \hat{F}^{a_1}(\hat{x}_1) \hat{F}^{a_2}(\hat{x}_2) \cdots \hat{F}^{a_N}(\hat{x}_N) \hat{\Phi} | 0 \rangle = 0, \quad (8)$$

where $\hat{F}^a = \partial^\mu \hat{A}_\mu^a + \xi \partial_5 \hat{A}_5^a$ is the 5d gauge-fixing function, and $\hat{\Phi}$ denotes other possible amputated (non-gauge) physical fields in 5d. After KK-expansion and amputation of the external fields in \hat{F}^a 's, we arrive at

$$\langle 0 | \bar{F}^{a_1 n_1}(k_1) \bar{F}^{a_2 n_2}(k_2) \cdots \bar{F}^{a_N n_N}(k_N) \Phi | 0 \rangle = 0, \quad (9)$$

where $\bar{F}^{an}(k) = ik^\mu A_\mu^{an} - C^{an} M_n^a A_5^{an} = iM_n^a (A_S^{an} + iC^{an} A_5^{an})$ with $A_S^{an} = \epsilon_S^\mu A_\mu^{an}$ ($\epsilon_S^\mu \equiv k^\mu/M_n^a$) and $C^{an} = 1 + O(\text{loop})$ ⁸. Our identity (9) just shows that the unphysical scalar-KK-component (A_S^{an}) and the 5th gauge-KK-component A_5^{an} are *confined* at the S -matrix level, so they together have zero contribution to any physical process. This is nothing but a quantitative formulation of the *Geometric Higgs Mechanism* (GHM) at the S -matrix level, where A_5^{an} serves as the geometric would-be Goldstone boson and gets converted to the physical longitudinal component A_L^{an} at each KK-level. The implications of this GHM are profound: (i) it ensures⁵ the nontrivial E -cancellations and the unitarity of $A_L^{an} A_L^{am}$ scatterings; (ii) it suggests^{5,6,7} the possibility that the electroweak symmetry breaking may be realized *geometrically* without physical Higgs boson^{9,10}. Noting that the $\hat{A}_L^a - \hat{A}_5^a$ conversion occurs under the 5d compactification, we require that the consistent BCs be imposed such that the action respects the 5d gauge symmetry (or the equivalent 5d BRST invariance) at the boundaries. We observe^{9,10} that all such consistent BCs can be reconstructed and classified from the continuum limits of proper gauge-invariant lattice formulation (deconstruction) of the extra dimensions.

Expanding the polarization vector $\epsilon_L^\mu = \frac{k^\mu}{M_n^a} + O\left(\frac{M_n^a}{E}\right)$, we have deduced, from the identity (9), the KK Equivalence Theorem (KK-ET)⁵,

$$\mathcal{T}[A_L^{a_1 n_1}, A_L^{a_2 n_2}, \cdots, \Phi] = C_{\text{mod}} \mathcal{T}[A_5^{a_1 n_1}, A_5^{a_2 n_2}, \cdots, \Phi] + O(M_n^a/E), \quad (10)$$

where $C_{\text{mod}} = (-i)^N [1 + O(\text{loop})]$. We stress that our above formulation of KK-ET (10)⁵ is completely general, *valid independent of whether the gauge group rank is preserved or reduced under compactification, i.e., independent of whether the zero-mode gauge fields remain massless or acquire KK-masses (or masses induced by usual Higgs)*. A crucial general observation⁵ is that the KK-ET ensures the nontrivial E -cancellations in the A_L^{an} -amplitude, as enforced by the GHM, because the A_5^{an} -amplitude on the RHS of (10) has all individual diagrams manifestly of $O(E^0)$ or smaller.

Hence, in the A_L^{an} -amplitude all terms of $O(E^q)$ ($q > 0$) must cancel down to $O(E^0)$. Our direct computation of the A_5^{an} -scattering in Fig. 2 gives,⁵

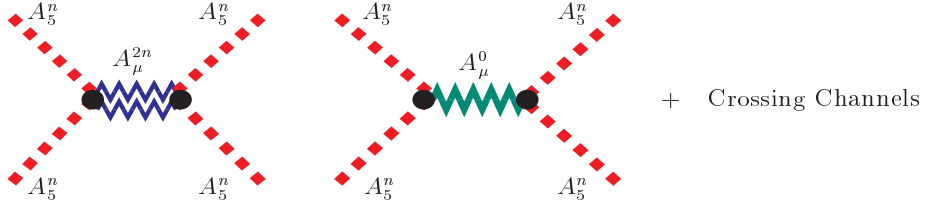


Figure 2. Goldstone scattering $A_5^{an} A_5^{an} \rightarrow A_5^{an} A_5^{an}$ in compactified KK theory.

$$T[A_5^{an} A_5^{bn} \rightarrow A_5^{cn} A_5^{dn}] = g^2 [C^{abe} C^{cde} \tilde{T}_1 + C^{ace} C^{dbe} \tilde{T}_2 + C^{ade} C^{bce} \tilde{T}_3],$$

$$\tilde{T}_1 = -\frac{3}{2}c, \quad \tilde{T}_2 = -\frac{3(3+c)}{2(1-c)}, \quad \tilde{T}_3 = \frac{3(3-c)}{2(1+c)}, \quad (11)$$

which is indeed *equivalent* to Eq. (7) after applying the Jacobi identity.

So far we have fully understood why the compactified higher-d gauge theory does exhibit low energy unitarity in all individual scattering channels. But, compactified KK theory is nonrenormalizable, and we observe⁵ that the KK scattering has to reflect the bad 5d high energy behavior in (1) via coupled channels because of a large number of KK states existing below the UV cutoff $\Lambda = N_{KK}/R$. In the gauge-singlet channel, $|\Psi\rangle = \frac{1}{\sqrt{N_0}} \sum_{\ell=0}^{N_0} |A_L^{a\ell} A_L^{a\ell}\rangle$, we find that the unitarity condition indeed cuts off the KK tower at $N_0 = N$ such that

$$\frac{N}{R} \lesssim \frac{\sqrt{32\pi}}{k} \frac{O(1)}{\tilde{g}^2} = O\left(\frac{1}{\tilde{g}^2}\right), \quad (12)$$

reproducing the feature in (2). The unitarity in the deconstructed 5d gauge theory was first studied in Ref. [6]. Our approach to the geometric electroweak symmetry breaking (GEWSB) is discussed elsewhere^{9,10}.

3. Unitarity and Higher-d Gauge Unification

Extending the above generic results to the 5d SM, we derive⁷ strong limits on the highest KK-level N_{KK} , as well as the zero-mode Higgs mass m_H ,

$$\begin{aligned} \text{5d QCD: } N_{KK} &\leq 4, & (\text{for } \alpha_s \simeq 0.1); \\ \text{5d EW: } N_{KK} &\leq 11, & (\text{for } g = 2m_w/v); \\ m_H &< v\sqrt{16\pi/3} N_{KK}^{-1/2} \simeq 303 \text{ GeV}, & (\text{for } N_{KK} = 11). \end{aligned} \quad (13)$$

With all gauge bosons propagating in the bulk, many KK states will contribute to the gauge coupling running and accelerate the gauge unification¹². We apply the unitarity analysis to the minimal 5d SUSY GUT¹² and find that imposing the unitarity limit $N_{KK}^{EW} \leq 11$ suggests the GUT scale $M_G \geq 10^{14}$ GeV, as shown in Fig. 3. An extension to 5d GUTs broken by orbifolds¹³ can be similarly performed⁷. The 5d GUTs with nontrivial UV fixed points¹⁴ appear attractive to reconcile the unitarity constraint⁷.

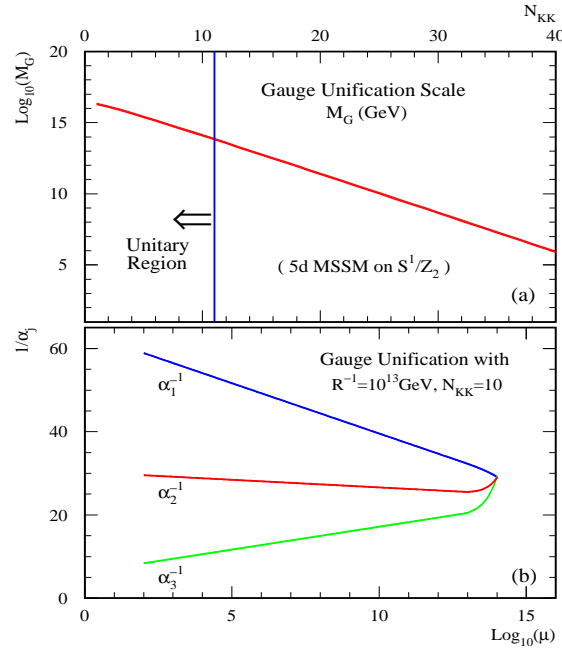


Figure 3. The 5d SUSY gauge unification.

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